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High School Students' Perceptions Of The Irrational Number Π

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Abstract –
Understanding irrational numbers is important to assimilate the concept of number. The number π (pi) is one of the most significant and fascinating irrational numbers. Note that π is a transcendental number. The present work aims to investigate how Moroccan students conceptualize the number π. The results obtained show that the largest part of students in the study have several misconceptions about some characteristics of the number π. Moreover, the analysis of these results allows us to propose some suitable techniques to detect students' conceptions in order to correct them during the learning processes.

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I. INTRODUCTION

One of the fundamental notions in mathematics, the concept of numbers appears in the early stages of education. According to Piaget [7], However, the representation of this concept which is not available at any age, requires a strong power of abstraction.

Recently, a number of educational research papers have been devoted to the study of the concept of numbers in many countries, especially in secondary education (see for instance, [4, 9, 10, 12]).

The number π is defined as the ratio of the circumference of a circle to its diameter. It can be also defined as the ratio of the area of a disc to the square of its radius. This number may be considered as the world's most popular mathematical constant that has fascinated so many scientists since ancient times. This constant appears in many formulas in mathematics and physics. Therefore, the number π was the source of many mathematical results [1, 11].

The irrationality of π was demonstrated by Lambert in 1761. This shows that π=3,141592…. has an infinite decimal expansion which is not periodic.

Several authors have recently studied high school students' perceptions of rational and irrational numbers (see [3, 5, 8]). The results obtained by Fischbein et al in [5] confirm that many students are unable to classify various numbers (rational, irrational, real). The purpose of the study, conducted by N. Sirotic and R. Zazkis [3], was to provide an account of teachers' understandings and misunderstandings of irrational numbers, to interpret how the understanding of irrationality is acquired, and to explain how and why difficulties occur. Kirdon [6] analyzed the way students approach the question of the existence of irrational numbers.

Based on a study on the place of the number in Moroccan textbooks from primary to secondary qualifying [2], we sought to identify and understand any gaps in the student's conception of the number PI. Our approach was to propose a questionnaire to a sample of 430 students in March 2019. The analysis of the questionnaire brought to light a poor knowledge of the main properties of π.

We present in the second section an overview of the methodology approach employed in the study, the target population and the questions raised. In the third section, we exhibit the results of our investigation.

II. RESEARCH CONTEXT AND METHODOLOGY

Working Approach:

To identify and analyze the student's representations about the number π, we conducted a survey of 430 students at the last year of the qualifying secondary school; the survey was carried out in April 2021 and covered nine establishments. Among them four schools were private institutions. The data collection method used was a list of multiple-choice questions including close-ended questions. We think that this type of questionnaire will give us more relevant information on how the learner's conceptions about the number π are constructed.

Study Population:

Our sample contains students in their last year of high school with a scientific specialization. The aim of this study is to improve the quality of education and to elaborate the scientific knowledge of our students. So, we randomly sampled 420 students from five public schools and four private schools.

III. RESULTS AND ANALYSIS

The questionnaire contains five specific and distinct parts. To analyze the results of the survey, we classified these responses in two categories; correct and incorrect answers. Then, we established a histogram to compare the responses of students according to the frequency of occurrence of each category. The responses are summarized in the table below.

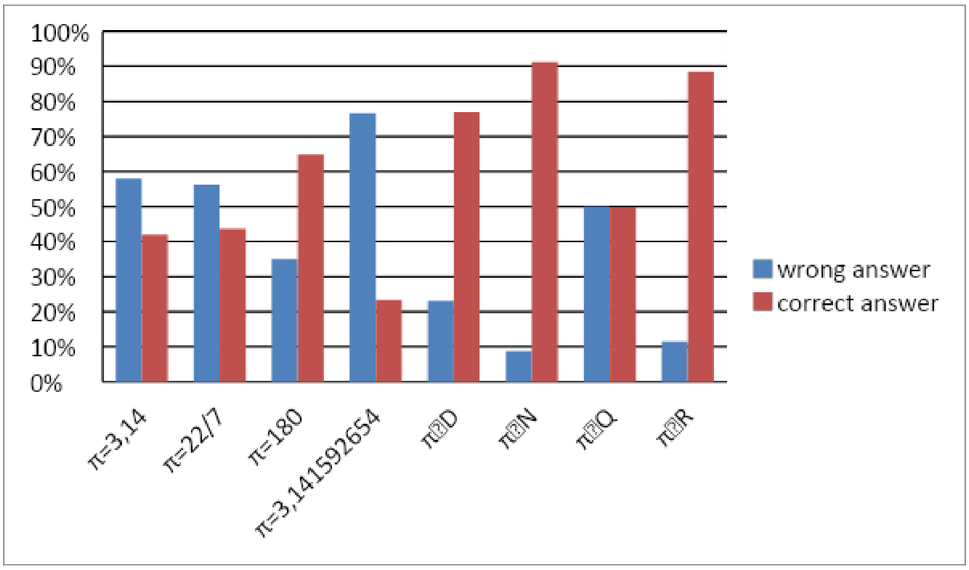
Complete by selecting true or false:

	True	**False**
$\pi = 3.14$		
$\pi = \frac{22}{7}$		
$\pi = 180$		
$\pi = 3.141592654$		
$\pi \in ID\ (\text{the set of decimal numbers})$		
$\pi \in IN\ (\text{the set of natural numbers})$		
$\pi \in Q\ (\text{the set of rational numbers})$		
$\pi \in IR\ (\text{the set of real numbers})$		

Tab.1: representation of number π

Results

We represent the student responses for the survy in the following graph:

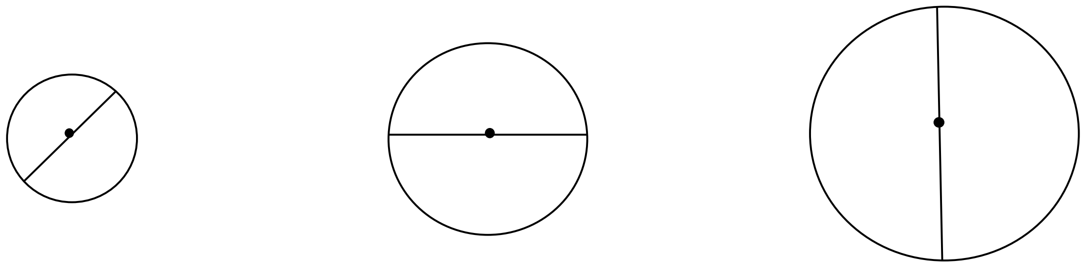


Graph: Answer's Students in of the survey

After analyzing the graphs, the three following point should be noted:

1. The most commonly used approximations of the value of π are 3.14 and 22/7. Both of these values are frequently used to calculate the circumference of a circle (or in physics). Therefore, even students at the Baccalaureate level forget that these values do not equal π and that π is an irrational number.
2. For half of the students in the sample, π is a rational number. Some of the incorrect answers can be due to an erroneous knowledge of the different sets IN, Z, ID, Q and IR.
3. Finally, more than a third of the students approved that π=180 which can be explained by an inaccurate understanding of the formula: π(rad)=180°

Note that for each k in {1, 2, 3}, the diameter of the circle (φ_k) is denoted by d_i.
Insert the appropriate symbol = or < or > in the blank spaces:



$$\frac{circumference\ of\ (\varphi_1)}{d_1} \ldots\ldots\ldots \frac{circumference\ of\ (\varphi_2)}{d_2} \ldots\ldots\ldots \frac{circumference\ of\ (\varphi_3)}{d_3}$$

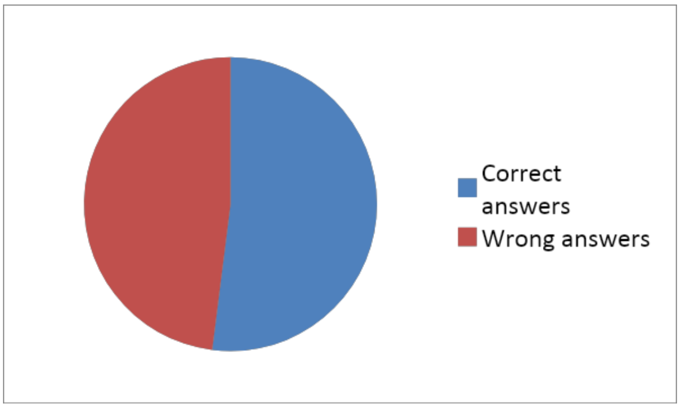
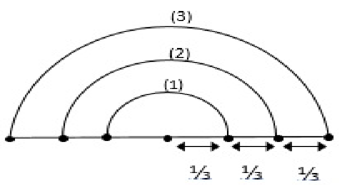


Fig.1: Results of the answers

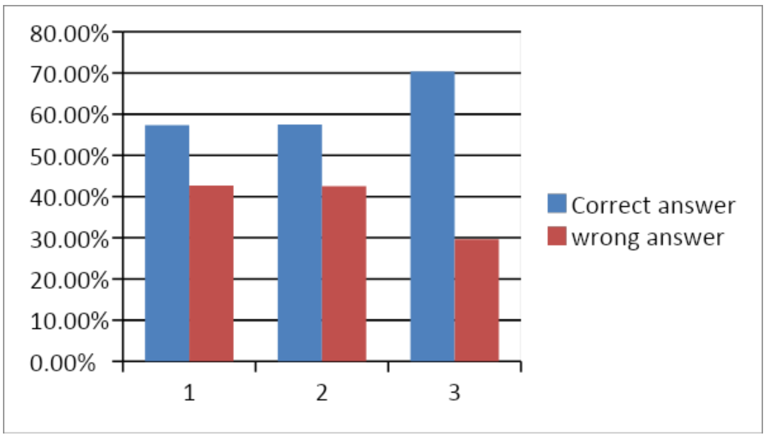
The answers to this question indicate that half of the students in the sample are unaware that π equals to the ratio between circumference of a circle and its diameter. This well-known result that is learnt in elementary school, is not fully retained during the high school years.

Insert one of the following two symbols in the blank to make each statement true: "=" or "≠" .



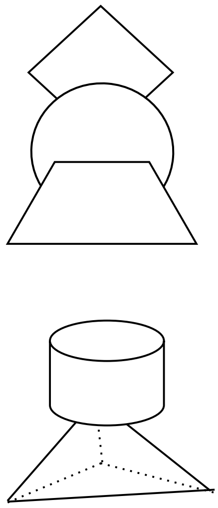
Arc length (1) ……….. π cm
Arc length (2) ……….. $\pi\ cm$
Arc length (3) ……….. $\pi\ cm$

<u>Results:</u>



We can estimate that around 60% of the students have a good understanding of the formula that states that the circumference of a circle is equal to π multiplied by its diameter.

Check the box corresponding to the geometric shape related to the number π:



Lozenge ☐

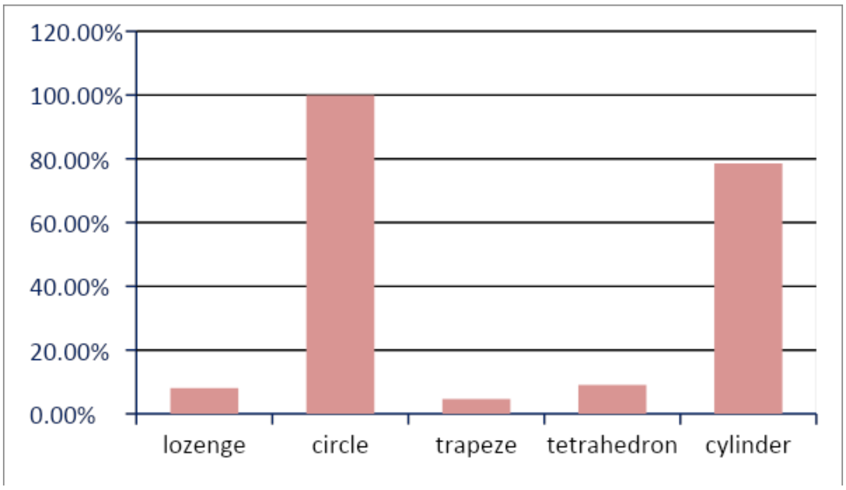
Circle ☐

Trapeze ☐

Tetrahedron ☐

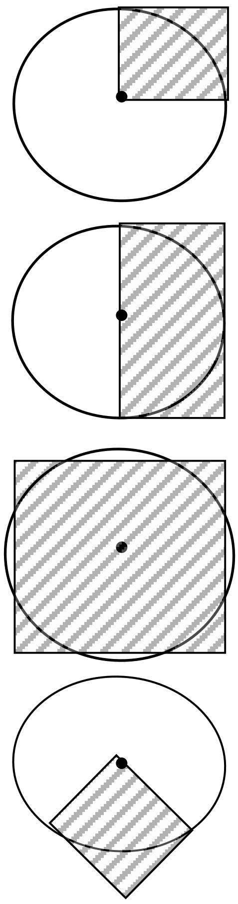
Cylinder ☐

Results:

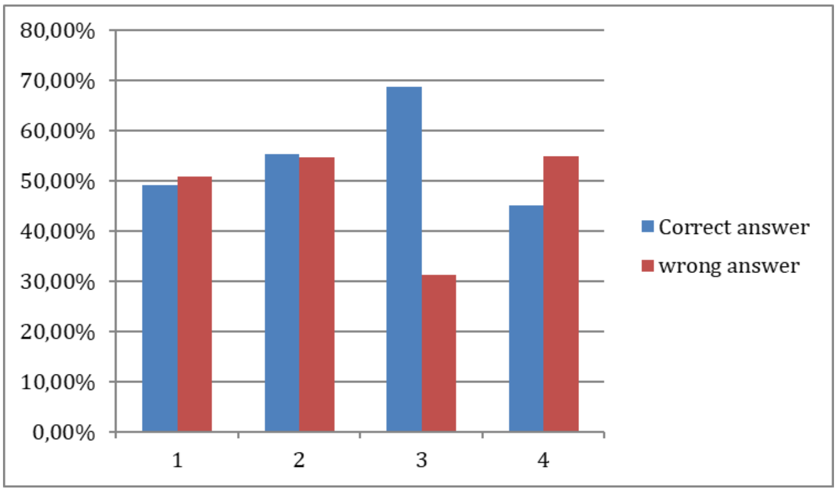


The main conclusion that can be drawn from this question is that all students seem to link the number π to the circle, and the large majority of participants connected the number π with the cylinder.

Complete with one of the following two symbols "=" or "≠".



Results



The answers to this question reveal that approximately half of the students in the sample didn't assimilate that π is the ratio of the area of a disc to the square of its radius.

Remark: An additional part of this study was devoted to investigate teachers' understanding of the number π. This part was conducted on 136 secondary school mathematics student teachers in 2022. The following question served as a research tool:

The number π is equal to :			
A	**B**	**C**	**D**
$\frac{22}{7}$	3,14	3,1415	Another answer

We were interested in examining whether future teachers are aware of the irrationality of the number π. Around 66,91% of the student teachers in the sample ignore that "*π is an irrational number*".

IV. CONCLUSION

According to our sample, students in the final phase of high school have difficulties in understanding certain properties of the number π. The most striking one is the misconception of the irrationality of this number by confusing it approximate values such as 3.14 or 22/7. This can be explained by the abundant use of these values in numerical calculations using formulas containing π.

It should also be noted that almost half the participants in this study were unable to correctly recall the geometric formulas involving the number π. On the other hand, the pupils showed a correct knowledge of the existing relation between the number π and the geometric circular shapes

Our findings can be explained by, among other factors, the results of the investigation done by Bettioui et all [2]. In fact, after analyzing mathematics school textbooks from primary to qualifying secondary school, the authors reported the following three findings:

1. During the primary school, students learned how to calculate the circumference of a circle and the area of a disc using the approximate value 3.14 for π.
2. There is a lack of activities related to π in middle schools. Therefore, it becomes harder for students to recall the concerned formulas and properties.
3. In high school the number, π appears in another context associated with measurement of angles.

In conclusion, finally the lack of consistency in the use of the number π throughout the first three cycles of teaching hinders the establishment of an intelligent understanding and a solid representation of this number. The logical consequence of this kind of progression is the withering away and loss of academic achievements when moving from one level to the next one. No doubt that this strategy advocated by school programs in terms of teaching the number π as the strategy of the fish biting its tail knowing that this number is used in several fields of scientific knowledge. Therefore, we propose to consider π among the pedagogical objectives of mathematics education and to integrate activities around this number at all levels with the mathematical notions acquired by the students and the recommended didactic tools, in order to keep a correct representation of this number.

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